

## THE COMPLETION OF GENERALIZED B-METRIC SPACES AND FIXED POINTS

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**Abstract.** We introduce the notion of generalized b-metric, as a b-metric which can take infinite values, and prove the existence and uniqueness of the completion of some particular b-metric spaces (called generalized strong b-metric spaces). Some fixed point results in b-metric spaces and their counterparts in generalized b-metric spaces are proved.

**Key Words and Phrases:** Metric space, metrizability, b-metric space, generalized b-metric space, completion of a generalized b-metric space, fixed point.

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