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A NEW ITERATIVE ALGORITHM FOR A GENERALIZED MIXED EQUILIBRIUM PROBLEM AND A COUNTABLE FAMILY OF NONEXPANSIVE-TYPE MAPS WITH APPLICATIONS

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Abstract. Let C be a nonempty closed and convex subset of a uniformly smooth and uniformly convex real Banach space with dual space E^*. In this paper, a new iterative algorithm of Krasnoselskii-type is constructed and used to approximate a common element of a generalized mixed equilibrium problem and a common fixed point of a countable family of generalized-J-nonexpansive maps. Applications of our theorem, in the case of real Hilbert spaces, complement and extend the results of Peng and Yao, (Taiwanese Journal of Mathematics Vol. 12, No. 6, pp. 1401-1432, September 2008); Nakajo and Takahashi, (J. Math. Anal. Appl. 273 (2003) 372-379); Martinez-Yanes and Xu, (Nonlinear Anal., 64 (2006), 2400-2411); Qin and Su, (J. Syst. Sci. and Complexity 21(2008) 474-482)

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