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ZERO POINT PRINCIPLE OF BALL-NEAR IDENTITY OPERATORS AND APPLICATIONS TO IMPLICIT OPERATOR PROBLEM

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Abstract. In this paper we give a global zero point principle for operators on a Banach space in terms of ball-near identity operator condition. The techniques of the proof are some variants of the saturated contraction principle. So, we study the well posedness of zero point problem, the Ostrowski property, data dependence and Ulam stability of zero point equations. Some relevant examples are given. Applications to the implicit operator problem are also presented.
Key Words and Phrases: Banach space, ball-near identity operator, saturated contraction principle, zero point principle, well posedness of zero point problem, Ostrowski property, data dependence, Ulam stability of zero point equations, implicit function problem, implicit operator problem.
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