

## AN IMPROVEMENT OF ALTERNATING DIRECTION METHOD FOR SOLVING VARIATIONAL INEQUALITY PROBLEMS WITH SEPARABLE STRUCTURE

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**Abstract.** In this paper, we propose a new modified alternating direction method for solving a class of variational inequalities with separable structure. The proposed method uses a new searching direction which differs from the others existing in the literature. The global convergence of the proposed method is studied under certain assumptions. Several special cases are also discussed. The results presented in this paper extend and improve some well-known results in the literature. We also reported some numerical results to illustrate the effectiveness and superiority of the proposed method.

**Key Words and Phrases:** Variational inequalities, monotone operator, projection method, alternating direction method.

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