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ABSTRACT MEASURES OF NONCOMPACTNESS AND FIXED POINTS FOR NONLINEAR MAPPINGS

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Abstract. In this paper, we study the existence of fixed points for a mapping by using abstract measures of noncompactness. Thus, we can obtain some generalizations of Darbo and Sadovskiĭ's theorems and we also give a characterization for the existence of fixed points of a mapping which is not necessarily continuous. Finally, we solve an open problem proposed by I.A. Rus in 2001.
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