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PICARD SEQUENCES IN b-METRIC SPACES

SUZANA ALEKSIĆ*, ZORAN D. MITROVIĆ** AND STOJAN RADENOVIĆ***

*University of Kragujevac, Faculty of Science, Department of Mathematics and Informatics,
Radoja Domanovića 12, 34000 Kragujevac, Serbia
E-mail: suzanasimic@kg.ac.rs

**University of Banja Luka, Faculty of Electrical Engineering,
Patre 5, 78000 Banja Luka, Bosnia and Herzegovina
E-mail: zoran.mitrovic@etf.unibl.org

***King Saud University, College of Science, Department of Mathematics,
Riyadh 11451, Saudi Arabia
E-mail: stojan.radenovic@tdtu.edu.vn

Abstract. The purpose of this article is to provide much simpler and shorter proofs of some important results in the framework of b-metric spaces. Namely, we show that the given contractive condition implies b-Cauchyness of the corresponding Picard sequence. The obtained results improve well-known comparable results in the literature.
Key Words and Phrases: b-metric space, b-complete, b-Cauchy, b-continuous, Picard sequence.
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