

## UNIFIED RELATION-THEORETIC FIXED POINT RESULTS VIA $F_{\mathcal{R}}$ -SUZUKI-CONTRACTIONS WITH AN APPLICATION

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**Abstract.** In this paper, we introduce the notion of  $F_{\mathcal{R}}$ -Suzuki-contraction where  $\mathcal{R}$  stands for an arbitrary binary relation and utilize the same to establish some existence and uniqueness fixed point results on metric spaces (not necessarily complete) equipped with arbitrary relation. Our results generalize, extend and unify several results of the existing literature. We also provide some examples to demonstrate the generality of our results. As an application of our main results, the existence and uniqueness of solution of a family of nonlinear matrix equations is discussed.

**Key Words and Phrases:** Complete metric spaces, binary relations, Suzuki-contraction mappings, fixed point.

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