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NONLINEAR ITERATION METHOD FOR MONOTONE VARIATIONAL INEQUALITY AND FIXED POINT PROBLEM

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Abstract. Using the subgradient extragradient and Halpern methods, we prove two strong convergence results for finding a solution of a variational inequality problem involving Lipschitz continuous monotone with the Lipschitz constant unknown and the solution is also a fixed point a quasi-nonexpansive mapping in real Hilbert space.
Key Words and Phrases: Monotone mappings, subgradient extragradient method, Halpern method, quasi-nonexpansive mapping, strong convergence, fixed point, Hilbert spaces.
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