

## COMMON FIXED POINT THEOREMS IN C\*-ALGEBRA-VALUED B-METRIC SPACES WITH APPLICATIONS TO INTEGRAL EQUATIONS

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**Abstract.** Based on the concepts of C\*-algebra-valued b-metric space, we give some common fixed point results in C\*-algebra-valued b-metric space. As an application, existence and uniqueness result for one type of integral equations is also discussed.

**Key Words and Phrases:** Fixed point, b-metric space, C\*-algebra, common fixed point, compatible, weakly compatible.

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