

FIXED POINT THEOREMS INVOLVING 1-SET-WEAKLY CONTRACTIVE OPERATORS IN WC-BANACH ALGEBRAS

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Abstract. In this paper, we establish some fixed point theorems for the sum and the product of nonlinear weakly sequentially continuous operators acting on a WC-Banach algebra and involving 1-set-weakly contractive maps. The obtained results are applied to a nonlinear functional integral equation in a suitable Banach algebra. Our results are extensions of several earlier results.

Key Words and Phrases: WC-Banach algebra, fixed point theorems, weak topology, measure of weak noncompactness, 1-set-weakly contractive.

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