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A DOMAIN-THEORETIC BISHOP-PHELPS THEOREM

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Abstract. In this paper, the notion of c-support points of a set in a semitopological cone is introduced. It is shown that any nonempty convex Scott closed bounded set has a c-support point in a cancellative bd-cone under certain condition. We also introduce the notion of wd-cone and then we prove a Bishop-Phelps type theorem for wd-cones, especially for normed cones, under appropriate conditions. Finally, using of the Bishop-Phelps technique, we obtain a result about the fixed points of a mapping on s-cones.
Key Words and Phrases: s-cone, Scott topology, support point, Bishop-Phelps theorem.
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