Fixed Point Theory, 20(2019), No. 2, 483-506
DOI: 10.24193/fpt-ro.2019.2.31
http://www.math.ubbcluj.ro/~nodeacj/sfptcj.html

NIELSEN THEORY ON INFRA-NILMANIFOLDS MODELED ON THE GROUP OF UNI-TRIANGULAR MATRICES

YOUNGGI CHOI*,[1], JONG BUM LEE**,[2] AND KYUNG BAI LEE***

*Department of Mathematics Education, Seoul National University
Seoul 08826, Korea
E-mail: yochoi@snu.ac.kr

**Department of Mathematics, Sogang University
Seoul 04107, Korea
E-mail: jlee@sogang.ac.kr

***Department of Mathematics, University of Oklahoma
Norman, OK 73019, USA
E-mail: kblee@math.ou.edu

Abstract. Let Nil_m be the group of $m \times m$ uni-triangular matrices. Then it is a connected and simply connected $(m-1)$-step nilpotent Lie group. Using the averaging formulas, we compute the spectra of the Lefschetz, Nielsen and Reidemeister (coincidence) numbers of maps on infra-nilmanifolds modeled on Nil_m. As a byproduct, we prove that the Bieberbach groups of Nil_m $(m \geq 4)$ with Γ_m as its nil-radical satisfy the R_∞ property.
Key Words and Phrases: Averaging formula, infra-nilmanifold, Lefschetz number, Nielsen number, Reidemeister number, uni-triangular matrix.
2010 Mathematics Subject Classification: 57S30, 57S20, 22E25.

References

[1] R.F. Brown, *On a homotopy converse to the Lefschetz fixed point theorem*, Pacific J. Math., **17**(1966), 407-411.
[2] Y. Cao, Z. Tan, *Automorphisms of the Lie algebra of strictly upper triangular matrices over a commutative ring*, Linear Algebra Appl., **360**(2003), 105-122.
[3] Y. Choi, J.B. Lee, K.B. Lee, *Infra-nilmanifolds modeled on the group of uni-triangular matrices*, Geom. Dedicata, 189(2017), 161-176.
[4] K. Dekimpe, P. Igodt, S. Kim, K.B. Lee, *Affine structures for closed* 3*-dimensional manifolds with* Nil*-geometry*, Quart. J. Math. Oxford Ser. (2), **46**(1995), 141-167.
[5] A. Fel'shtyn, J.B. Lee, *The Nielsen and Reidemeister numbers of maps on infra-solvmanifolds of type* (R), Topology Appl., **181**(2015), 62-103.

[1]The first-named author was supported by Basic Science Research Program through the National Research Foundation of Korea(NRF) (NRF-2013R1A1A2006926).
[2]The second-named author was supported by Basic Science Researcher Program through the National Research Foundation of Korea(NRF) (NRF-2016R1D1A1B01006971).

[6] K.Y. Ha, J.B. Lee, P. Pennincks, *Formulas for the Reidemeister, Lefschetz and Nielsen coincidence number of maps between infra-nilmanifolds*, Fixed Point Theory Appl., **39**(2012), 1-23.
[7] S.W. Kim, J.B. Lee, *Averaging formula for Nielsen coincidence numbers*, Nagoya Math. J., **186**(2007), 69-93.
[8] S.W. Kim, J.B. Lee, K.B. Lee, *Averaging formula for Nielsen numbers*, Nagoya Math. J., **178**(2005), 37-53.
[9] K.B. Lee, *Maps of infra-nilmanifolds*, Pacific J. Math., **168**(1995), no. 1, 157-166.
[10] J.B. Lee, K.B. Lee, *Lefschetz numbers for continuous maps, and periods for expanding maps on infra-nilmanifolds*, J. Geom. Phys., **56**(2006), 2011-2023.
[11] M. Newman, *Integral Matrices*, Pure and Applied Mathematics, Vol. 45, Academic Press, New York-London, 1972.

Received: December 21, 2016; Accepted: May 18, 2017.