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HILFER AND HADAMARD RANDOM FRACTIONAL DIFFERENTIAL EQUATIONS IN FRÉCHET SPACES

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Abstract. This paper deals with some existence and Ulam stability results for Hilfer and Hilfer-Hadamard type fractional random differential equations in Fréchet spaces. A random fixed point theorem is applied to prove the existence of random solutions. Also it is shown that the solutions to our problems are generalized Ulam-Hyers-Rassias stable.
Key Words and Phrases: Functional random differential equation, Riemann-Liouville integral of fractional order, Hadamard integral of fractional order, Hilfer fractional derivative, Hilfer-Hadamard fractional derivative, Fréchet space, Ulam stability, random fixed point.
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