

THE FIXED POINT PROPERTY FOR CLOSED NEIGHBORHOODS OF LINE SEGMENTS IN L^p

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Abstract. We prove that, in L^p -spaces with $p \in (1, \infty]$, closed neighborhoods of line segments are dismantlable and hence every monotone operator on these neighborhoods has a fixed point. We also give an example that, for $p = 1$, closed neighborhoods of line segments need not be dismantlable. It is an open question whether every monotone self map of a closed neighborhood of a line segment in L^1 has a fixed point.

Key Words and Phrases: Dismantlable ordered set, fixed point property, line segment, closed L^p -neighborhood.

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