REMARKS ON FRINK'S METRIZATION TECHNIQUE AND APPLICATIONS

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Abstract. In this paper, we give a simple counterexample to show again the limits of Frink's construction [17, page 134] and then use Frink's metrization technique to answer two conjectures posed by Berinde and Choban [5], and to calculate corresponding metrics induced by some b-metrics known in the literature. We also use that technique to prove a metrization theorem for 2-generalized metric spaces, and to deduce the Banach contraction principle in b-metric spaces and 2-generalized metric spaces from that in metric spaces.
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References

[1] H. Aimar, B. Iaffei, and L. Nitti, *On the Macías-Segovia metrization of quasi-metric spaces*, Rev. Un. Mat. Argentina, **41**(1998), 67-75.

[2] T.V. An, N.V. Dung, Z. Kadelburg, Stojan Radenović, *Various generalizations of metric spaces and fixed point theorems*, Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat., **109**(2015), 175-198.

[3] T.V. An, L.Q. Tuyen, N.V. Dung, *Stone-type theorem on b-metric spaces and applications*, Topology Appl., **185-186**(2015), 50-64.

[4] A.V. Arhangel'skii, *Mappings and spaces*, Russian Math. Surveys, **21**(1966), 115-162.

[5] V. Berinde, M. Choban, *Generalized distances and their associate metrics. Impact on fixed point theory*, Creat. Math. Inform., **22**(2013), no. 1, 23-32.

[6] M. Boriceanu, *Fixed point theory for multivalued contractions on a set with two b-metrics*, Creat. Math. Inform., **17**(2008), no. 3, 326-332.

[7] M. Boriceanu, M. Bota, Adrian Petruşel, *Multivalued fractals in b-metric spaces*, Central Eur. J. Math., **8**(2010), no. 2, 367-377.

[8] M. Bota, A. Molnár, and C. Varga, *On Ekeland's variational principle in b-metric spaces*, Fixed Point Theory, **2011**(2011), 21-28.

[9] A. Branciari, *A fixed point theorem of Banach-Caccioppoli type on a class of generalized metric spaces*, Publ. Math. Debrecen, **57**(2000), no. 1-2, 31-37.

[10] E. W. Chittenden, *On the equivalence of ecart and voisinage*, Trans. Amer. Math. Soc., **18**(1917), no. 2, 161-166.

[11] E.W. Chittenden, *On the metrization problem and related problems in the theory of abstract sets*, Bull. Amer. Math. Soc., **33**(1927), 13-34.

[12] S. Czerwik, *Contraction mappings in b-metric spaces*, Acta Math. Univ. Ostrav., **1**(1993), no. 1, 5-11.

[13] S. Czerwik, *Nonlinear set-valued contraction mappings in b-metric spaces*, Atti Sem. Math. Fis. Univ. Modena, **46**(1998), 263-276.

[14] P. Das, L.K. Dey, *Fixed point of contractive mappings in generalized metric spaces*, Math. Slovaca, **59**(2009), no. 4, 499-504.

[15] R. Fagin, R. Kumar, D. Sivakumar, *Comparing top k lists*, SIAM J. Discrete Math., **17**(2003), no. 1, 134-160.

[16] S.P. Franklin, *Spaces in which sequences suffice*, Fund. Math., **57**(1965), 107-115.

[17] A.H. Frink, *Distance functions and the metrization problem*, Bull. Amer. Math. Soc., **43**(1937), no. 2, 133-142.

[18] M. Jovanović, Z. Kadelburg, S. Radenović, *Common fixed point results in metric-type spaces*, Fixed Point Theory Appl., **2010**(2010), 1-15.

[19] N. Kalton, *Quasi-Banach spaces*, Handbook of the geometry of Banach spaces 2 (W. B. Johnson and J. Lindenstrauss - eds.), Elsevier, 2003, 1099-1130.

[20] M.A. Khamsi, *Remarks on cone metric spaces and fixed point theorems of contractive mappings*, Fixed Point Theory Appl., **2010**(2010), 1-7.

[21] M.A. Khamsi, *Generalized metric spaces: A survey*, J. Fixed Point Theory Appl., **17**(2015), no. 3, 455-475.

[22] M.A. Khamsi, N. Hussain, *KKM mappings in metric type spaces*, Nonlinear Anal., **7**(2010), no. 9, 3123-3129.

[23] L. Kikina, K. Kikina, *On fixed point of a Ljubomir Ciric quasi-contraction mapping in generalized metric spaces*, Publ. Math. Debrecen, **83**(2013), no. 3, 1-6.

[24] W. Kirk, N. Shahzad, *Fixed point theory in distance spaces*, Springer, Cham, 2014.

[25] W.A. Kirk, N. Shahzad, *Generalized metrics and Caristi's theorem*, Fixed Point Theory Appl., **2013:129** (2013), 1-9.

[26] W.A. Kirk, N. Shahzad, *Correction: Generalized metrics and Caristi's theorem*, Fixed Point Theory Appl., **2014**(2014), 1-3.

[27] P. Kumam, N.V. Dung, *Remarks on generalized metric spaces in the Branciari's sense*, Sarajevo J. Math., **10**(2014), no. 2, 209-219.

[28] P. Kumam, N.V. Dung, V.T.L. Hang, *Some equivalences between cone b-metric spaces and b-metric spaces*, Abstr. Appl. Anal., **2013**(2013), 1-8.

[29] P. Lo'lo', S. M. Vaezpour, J. Esmaily, *Common best proximity points theorem for four mappings in metric-type spaces*, Fixed Point Theory Appl., **2015:47**(2015), 1-7.

[30] R.A. Macías, C. Segovia, *Lipschitz functions on spaces of homogeneous type*, Adv. Math., **33**(1979), no. 3, 257-270.

[31] M. Paluszyński, K. Stempak, *On quasi-metric and metric spaces*, Proc. Amer. Math. Soc., **137** (2009), no. 12, 4307-4312.

[32] A. Petruşel, I.A. Rus, *Fixed point theorems in ordered L-spaces*, Proc. Amer. Math. Soc., **134**(2006), no. 2, 411-418.

[33] A.B. Pilarska, T.D. Benavides, *The fixed point property for some generalized nonexpansive mappings and renormings*, J. Math. Anal. Appl., **429**(2015), no. 2, 800-813.

[34] S. Reich, A. J. Zaslavski, *Genericity and porosity in fixed point theory: a survey of recent results*, Fixed Point Theory Appl., **2015:195**(2015), 1-21.

[35] V. Schroeder, *Quasi-metric and metric spaces*, Conform. Geom. Dyn., **10**(2006), 355-360.

[36] T. Suzuki, *Generalized metric spaces do not have the compatible topology*, Abstr. Appl. Anal., **2014**(2014), 1-5.

[37] T. Suzuki, B. Alamri, L.A. Khan, *Some notes on fixed point theorems in ν-generalized metric spaces*, Bull. Kyushu Inst. Tech. Pure Appl. Math., **62**(2015), 15-23.

[38] T. Suzuki, B. Alamri, M. Kikkawa, *Only 3-generalized metric spaces have a compatible symmetric topology*, Open Math., **13**(2015), no. 1, 510-517.

[39] Q. Xia, *The geodesic problem in quasimetric spaces*, J. Geom. Anal., **19**(2009), 452-479.