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A NOTE ON EXISTENCE AND UNIQUENESS FOR INTEGRAL EQUATIONS WITH SUM OF TWO OPERATORS: PROGRESSIVE CONTRACTIONS

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Abstract. In this note we show a simple way to obtain a unique solution on $[0,\infty)$ of a scalar integral equation

$$x(t) = g(t, x(t)) + \int_0^t A(t-s)f(s, x(s))ds$$

where $x, y \in \Re$ and $t \geq 0$ imply that $|g(t,x) - g(t,y)| \leq \alpha|x-y|, 0 < \alpha < 1$, and for each $E > 0$ there is a $K > 0$ so that $x, y \in \Re$ and $0 \leq t \leq E$ imply $|f(t,x) - f(t,y)| \leq K|x-y|$. We introduce a *progressive contraction*. The constant K is a function of E and, hence, may tend to infinity as $E \to \infty$. The conclusion is that there is a single function $\xi(t)$ satisfying the equation on $[0,\infty)$ without resorting to any of the classical translations and extensions of solutions which, in fact, must invoke Zorn's Lemma and which can encounter difficulties as $K \to \infty$.
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