

## RELATION-THEORETIC METRICAL FIXED POINT THEOREMS UNDER NONLINEAR CONTRACTIONS

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**Abstract.** We establish fixed point theorems for nonlinear contractions on a metric space (not essentially complete) endowed with an arbitrary binary relation. Our results extend, generalize, modify and unify several known results especially those contained in Samet and Turinici [Commun. Math. Anal. 13, 82-97 (2012)] and Alam and Imdad [J. Fixed Point Theory Appl. 17(4), 693-702 (2015)]. Interestingly a corollary to one of our main results proved under symmetric closure of any binary relation remains a sharpened version of a theorem due to Samet and Turinici. Finally, we use examples to highlight the realized improvements in the results proved in this paper.

**Key Words and Phrases:** Complete metric spaces, binary relations, contraction mappings, fixed point.

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