POSITIVITY SOLUTIONS FOR A SYSTEM OF $p$-LAPLACIAN BOUNDARY VALUE PROBLEMS

JIAFA XU*, DONAL O’REGAN**,*** AND KEYU ZHANG****

*School of Mathematical Sciences, Chongqing Normal University, Chongqing 401331, China
E-mail: xujiafa292@sina.com and jiafaxu@sina.cn

**School of Mathematics, Statistics and Applied Mathematics, National University of Ireland Galway, Ireland

***NAAM Research Group, Department of Mathematics, King Abdulaziz University, Saudi Arabia
E-mail: donal.oregan@nuigalway.ie

****Department of Mathematics, Qilu Normal University, Jinan 250013, Shandong, China
E-mail: keyu292@163.com

Abstract. In this paper, we investigate the existence of positive solutions for a system of fourth order $p$-Laplacian boundary value problems

$$
\begin{cases}
-(|x'''|^{p-1})' = f(t,x,x',y,y'), & t \in [0,1],
-(|y'''|^{p-1})' = g(t,x,x',y,y'), & t \in [0,1],

x(0) = x'(1) = x''(0) = x'''(1) = 0,
y(0) = y'(1) = y''(0) = y'''(1) = 0,
\end{cases}
$$

where $p > 1$, $f,g \in C([0,1] \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+, \mathbb{R}^+ :=[0,\infty))$. Under some new general conditions on $f$ and $g$, we use the fixed point index to establish two existence theorems for the above system. The interesting point lies in the fact that the nonlinear term $f,g$ can be allowed to depend on the first derivative of the unknown functions, and this derivative dependence in systems is seldom considered in the literature.

Key Words and Phrases: $p$-Laplacian equation; positive solution; fixed point index; derivative dependence.

2010 Mathematics Subject Classification: 34B18, 47H07, 47H11, 45M20, 26D15.

Acknowledgement. This research is supported by National Science Fund for Young Scholars of China (Grant No.11601048), Natural Science Foundation of Chongqing (Grant No.cstc2016jcyjA0181), the Science and Technology Research Program of Chongqing Municipal Education Commission (Grant No.KJ1703050), Natural Science Foundation of Chongqing Normal University (Grant No.15XLB011).

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Received: August 28, 2014; Accepted: March 12, 2017.