

## FIXED POINTS, NORMAL STRUCTURE AND SLICES OF BANACH SPACES

SATIT SAEJUNG\* AND JI GAO\*\*,<sup>1</sup>

\*Department of Mathematics, Faculty of Science  
Khon Kaen University, Khon Kaen 40002, Thailand  
E-mail: saejung@kku.ac.th

\*\*Department of Mathematics, Community College of Philadelphia  
Philadelphia, PA 19130-3991, USA  
E-mail: jgao@ccp.edu

**Abstract.** In this paper, we first study the fixed point property for nonexpansive mappings of a Banach space and some existing result in [7] is extended. We secondly study the relationship between uniform normal structure and slices and some results in [11] are improved too.

**Key Words and Phrases:** Nonexpansive mapping, normal structure, ultraproduct, uniform normal structure, WORTH property.

**2010 Mathematics Subject Classification:** 46B20, 47H10, 37C25, 54H25.

### REFERENCES

- [1] A.G. Aksoy, M.A. Khamsi, *Nonstandard Methods in Fixed Point Theory*, Universitext, Springer-Verlag, New York, 1990.
- [2] M.S. Brodskii, D.P. Mil'man, *On the center of a convex set*, Doklady Akad. Nauk SSSR (N.S.), **59**(1948), 837–840.
- [3] D. Dacunha-Castelle, J.L. Krivine, *Applications des ultraproduits à l'étude des espaces et des algèbres de Banach*, Studia Math., **41**(1972), 315–334.
- [4] J. Diestel, *The Geometry of Banach Spaces – Selected Topics*, Lecture Notes in Math., Vol. 485, Springer Verlag, Berlin and New York, 1975.
- [5] H. Fetter, B. Gamboa de Buen, *Properties WORTH and WORTH\*,  $(1 + \delta)$  embeddings in Banach spaces with 1-unconditional basis and wFPP*, Fixed Point Theory Appl., 2010, Art. ID 342691, 7 pp.
- [6] K. Goebel, W.A. Kirk, *Topics in Metric Fixed Point Theory*, Cambridge Studies in Advanced Mathematics, 28. Cambridge University Press, Cambridge, 1990.
- [7] A. Jiménez-Melado, *The fixed point property for some uniformly nonoctahedral Banach spaces*, Bull. Austral. Math. Soc., **59**(1999), 361–367.
- [8] W.A. Kirk, *A fixed point theorem for mappings which do not increase distances*, Amer. Math. Monthly, **72**(1965), 1004–1006.
- [9] P.K. Lin, *Unconditional bases and fixed points of nonexpansive mappings*, Pacific J. Math., **116**(1985), 69–76.

---

<sup>1</sup>Corresponding author.

- [10] S. Saejung, *Convexity conditions and normal structure of Banach spaces*, J. Math. Anal. Appl., **344**(2008), 851–856.
- [11] S. Saejung, J. Gao, *Normal structure, slices and other properties in Banach spaces*, WSEAS Trans. on Mathematics, **11**(2012), 1094–1102.
- [12] B. Sims, "Ultra"-techniques in Banach Space Theory, Queen's Papers in Pure and Applied Mathematics, 60, Queen's University, Kingston, ON, 1982.
- [13] B. Sims, A class of spaces with weak normal structure, Bull. Austral. Math. Soc., **49**(1994), 523–528.

Received: August 22, 2014; Accepted: January 8, 2015.

