

ON SEMILINEAR FRACTIONAL ORDER DIFFERENTIAL INCLUSIONS IN BANACH SPACES

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Abstract. We are considering the Cauchy problem for a semilinear fractional differential inclusion in a Banach space. By using the fixed point theory for condensing multivalued maps, we prove the local and global theorems of the existence of mild solutions to this problem. We verify the compactness of the solutions set and its continuous dependence on parameters and initial data. We demonstrate also the application of the averaging principle to the investigation of the continuous dependence of the solutions set on a parameter in the case when the right-hand side of the inclusion is rapidly oscillating.

Key Words and Phrases: Fractional differential inclusion, semilinear differential inclusion, Cauchy problem, continuous dependence of solutions, averaging principle, fixed point, multivalued map, condensing map, measure of noncompactness.

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