

A FIXED POINT THEOREM OF MARKOV-KAKUTANI TYPE FOR A COMMUTING FAMILY OF CONVEX MULTIVALUED MAPS

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Abstract. Let Γ be a commuting family of upper semicontinuous convex multivalued maps of K into itself with nonempty closed values, where K is a nonempty compact convex subset of a locally convex Hausdorff topological vector space E . We then show that the Markov-Kakutani fixed-point theorem holds; that is, there exists at least one point $x \in K$ such that $x \in u(x)$ for all u in Γ .

Key Words and Phrases: fixed-point theorem, multimaps, invariant measure of multimaps, locally convex Hausdorff topological space.

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