Fixed Point Theory, 18(2017), No. 1, 137-146
http://www.math.ubbcluj.ro/~nodeacj/sfptcj.html

APPROXIMATING FIXED POINTS OF THE COMPOSITION OF TWO RESOLVENT OPERATORS

OGANEDITSE A. BOIKANYO

Department of Mathematics and Statistical Sciences Botswana
International University of Science and Technology Private Bag 16, Palapye, Botswana
E-mail: boikanyoa@gmail.com

Abstract. Let A and B be maximal monotone operators defined on a real Hilbert space H, and let $\mathrm{Fix}(J_\mu^A J_\mu^B) \neq \emptyset$, where $J_\mu^A y := (I + \mu A)^{-1} y$ and μ is a given positive number. [H. H. Bauschke, P. L. Combettes and S. Reich, The asymptotic behavior of the composition of two resolvents, Nonlinear Anal. 60 (2005), no. 2, 283-301] proved that any sequence (x_n) generated by the iterative method $x_{n+1} = J_\mu^A y_n$, with $y_n = J_\mu^B x_n$ converges weakly to some point in $\mathrm{Fix}(J_\mu^A J_\mu^B)$. In this paper, we show that the modified method of alternating resolvents introduced in [O. A. Boikanyo, A proximal point method involving two resolvent operators, Abstr. Appl. Anal. 2012, Article ID 892980, (2012)] produces sequences that converge strongly to some points in $\mathrm{Fix}(J_\mu^A J_\mu^B)$ and $\mathrm{Fix}(J_\mu^B J_\mu^A)$.
Key Words and Phrases: Maximal monotone operator, alternating resolvents, proximal point algorithm, nonexpansive map, resolvent operator.
2010 Mathematics Subject Classification: 47J25, 47H05, 47H09, 47H10.

Acknowledgment. The author would like to sincerely thank Professor Simeon Reich for his valuable comments that improved the presentation of this manuscript.

References

[1] H.H. Bauschke, P.L. Combettes, S. Reich, *The asymptotic behavior of the composition of two resolvents*, Nonlinear Anal., **60**(2005), no. 2, 283-301.
[2] H.H. Bauschke, E. Matoušková, S. Reich, *Projection and proximal point methods: convergence results and counterexamples*, Nonlinear Anal., **56**(2004), no. 5, 715-738.
[3] O.A. Boikanyo, *A proximal point method involving two resolvent operators*, Abstr. Appl. Anal. 2012, Article ID 892980, 2012, 12 p.
[4] O.A. Boikanyo, G. Moroşanu, *Inexact Halpern-type proximal point algorithm*, J. Glob. Optim., **51**(2011), no. 1, 11-26.
[5] O.A. Boikanyo, G. Moroşanu, *A contraction proximal point algorithm with two monotone operators*, Nonlinear Anal., **75**(2012), no. 14, 5686-5692.
[6] O.A. Boikanyo, G. Moroşanu, *On the method of alternating resolvents*, Nonlinear Anal., **74**(2011), 5147-5160.
[7] O.A. Boikanyo, G. Moroşanu, *Strong convergence of the method of alternating resolvents*, J. Nonlinear Convex Anal., **14**(2013), no. 2, 221-229.
[8] L.M. Bregman, *The method of successive projection for finding a common point of convex sets*, Sov. Math. Dokl., **6**(1965), 688-692.
[9] L. Hu, L. Liwei, *A new iterative algorithm for common solutions of a finite family of accretive operators*, Nonlinear Anal., **70**(2009), 2344-2351.

[10] H. Hundal, *An alternating projection that does not converge in norm*, Nonlinear Anal., **57**(2004), no. 1, 35-61.

[11] S. Kamimura, W. Takahashi, *Approximating solutions of maximal monotone operators in Hilbert spaces*, J. Approx. Theory, **106**(2000), 226–240.

[12] E. Kopecká, S. Reich, *A note on the von Neumann alternating projections algorithm*, J. Nonlinear Convex Anal., **5**(2004), 379-386.

[13] E. Matoušková, S. Reich, *The Hundal example revisited*, J. Nonlinear Convex Anal., **4**(2003), 411-427.

[14] N. Lehdili, A. Moudafi, *Combining the proximal algorithm and Tikhonov regularization,* Optimization, **37**(1996), 239-252.

[15] G. López, V. Martín-Márquez, F. Wang, H.K. Xu, *Forward-Backward Splitting Methods for Accretive Operators in Banach Spaces*, Abstr. Appl. Anal. 2012, Article ID 109236, 2012, 25 p.

[16] P.E. Maingé, *Strong convergence of projected subgradient methods for nonsmooth and nonstrictly convex minimization*, Set-Valued Anal., **16**(2008), 899-912.

[17] G. Moroşanu, *Nonlinear Evolution Equations and Applications*, Reidel, Dordrecht, 1988.

[18] O. Nevanlinna, S. Reich, *Strong convergence of contraction semigroups and of iterative methods for accretive operators in Banach spaces*, Israel J. Math., **32**(1979), 44-58.

[19] F. Wang, H. Cui, *On the contraction-proximal point algorithms with multi-parameters*, J. Glob. Optim., **54**(2012), 485-491.

[20] H.K. Xu, *Iterative algorithms for nonlinear operators,* J. London Math. Soc., **66**(2002), no. 2, 240-256.

[21] H.K. Xu, *A regularization method for the proximal point algorithm,* J. Glob. Optim., **36**(2006), 115-125.

[22] H. Zegeye, N. Shahzad, *Strong convergence theorems for a common zero of a finite family of m-accretive mappings*, Nonlinear Anal., **66**(2007), 1161-1169.

[23] H. Zegeye, N. Shahzad, *Proximal point algorithms for finding a zero of a finite sum of monotone mappings in Banach spaces*, Abstr. Appl. Anal. 2013, Article ID 232170, 2013, 7 p.

Received: April 25, 2014; Accepted: October 23, 2014.