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ON A FIXED POINT THEOREM IN UNIFORM SPACES AND ITS APPLICATION TO NONLINEAR VOLTERRA TYPE OPERATORS

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Abstract. In the present work, we prove a fixed point theorem for nonlinear operators acting in Hausdorff sequentially complete uniform spaces whose uniformity is generated by a saturated family of pseudometrics. As an application we consider nonlinear abstract Volterra type integral equations of second kind in the case when the independent variable belongs to arbitrary completely regular Hausdorff space. Existence and uniqueness of the solutions of these equations in nonhomogeneous case are also proved.
Key Words and Phrases: Fixed point, Volterra type integral equations, uniform space, pseudometrics, Hausdorff space.
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