

## SOME STABILITY CONCEPTS FOR ABSTRACT FRACTIONAL DIFFERENTIAL EQUATIONS WITH NOT INSTANTANEOUS IMPULSES

SAÏD ABBAS\*, MOUFFAK BENCHOHRA\*\*, AHMED ALSAEDI\*\*\*,1  
AND YONG ZHOU\*\*\*\*,1

\*Laboratory of Mathematics, University of Saïda,  
PO Box 138, 20000 Saïda, Algeria  
E-mail: [abbasmsaid@yahoo.fr](mailto:abbasmsaid@yahoo.fr)

\*\*Laboratory of Mathematics, University of Sidi Bel-Abbes,  
P.O. Box 89, Sidi Bel-Abbès 22000, Algeria  
E-mail: [benchohra@yahoo.com](mailto:benchohra@yahoo.com)

\*\*\*Department of Mathematics, Faculty of Science, King Abdulaziz University  
P.O. Box 80203, Jeddah 21589, Saudi Arabia

\*\*\*\*Department of Mathematics, Xiangtan University  
Hunan 411105, PR China  
E-mail: [yzhou@xtu.edu.cn](mailto:yzhou@xtu.edu.cn)

**Abstract.** In this paper, we investigate some uniqueness and Ulam's type stability concepts for functional abstract fractional differential equations with not instantaneous impulses in Banach spaces.

**Key Words and Phrases:** abstract fractional differential equation, mild solution, impulse, Ulam-Hyers-Rassias stability, fixed point theorems.

**2010 Mathematics Subject Classification:** 26A33, 34A37, 34D10, 47H10.

**Acknowledgement.** The third author's work was supported by National Natural Science Foundation of P.R. China (11271309), the Specialized Research Fund for the Doctoral Program of Higher Education (20114301110001) and Hunan Provincial Natural Science Foundation of China (12JJ2001).

### REFERENCES

- [1] S. Abbas, M. Benchohra, *Fractional order partial hyperbolic differential equations involving Caputo's derivative*, Stud. Univ. Babeş-Bolyai Math., **57**(2012), no. 3, 469-479.
- [2] S. Abbas, M. Benchohra, *Ulam-Hyers stability for the Darboux problem for partial fractional differential and integro-differential equations via Picard operators*, Results. Math., **65**(2014), no. 1-2, 67-79.
- [3] S. Abbas, M. Benchohra, L. Gorniewicz, *Existence theory for impulsive partial hyperbolic functional differential equations involving the Caputo fractional derivative*, Sci. Math. Jpn., online 2010, 271-282.

---

<sup>1</sup>Corresponding author.

- [4] S. Abbas, M. Benchohra, J. Henderson, *Asymptotic attractive nonlinear fractional order Riemann-Liouville integral equations in Banach algebras*, Nonlinear Studies, **20**(2013), no. 1, 1-10.
- [5] S. Abbas, M. Benchohra, G.M. N'Guérékata, *Topics in Fractional Differential Equations*, Developments in Mathematics, 27, Springer, New York, 2012.
- [6] S. Abbas, M. Benchohra, J.J. Nieto, *Ulam stabilities for impulsive partial fractional differential equations*, Acta Univ. Palacki. Olomuc, **53** (2014), 5-17.
- [7] S. Abbas, M. Benchohra, A.N. Vityuk, *On fractional order derivatives and Darboux problem for implicit differential equations*, Frac. Calc. Appl. Anal., **15**(2012), no. 2, 168-182.
- [8] S. Abbas, M. Benchohra, Y. Zhou, *Darboux problem for fractional order neutral functional partial hyperbolic differential equations*, Int. J. Dyn. Syst. Differ. Eq., **2**(2009), no. 3-4, 301-312.
- [9] M. Benchohra, J.R. Graef, S. Hamani, *Existence results for boundary value problems of nonlinear fractional differential equations with integral conditions*, Appl. Anal., **87**(2008), no. 7, 851-863.
- [10] K. Diethelm, N.J. Ford, *Analysis of fractional differential equations*, J. Math. Anal. Appl., **265**(2002), 229-248.
- [11] M.M. El-Borai, *Some probability densities and fundamental solutions of fractional evolution equations*, Chaos Solitons & Fractals, **14**(2002), 433-440.
- [12] D. Henry, *Geometric Theory of Semilinear Parabolic Partial Differential Equations*, Springer-Verlag, Berlin-New York, 1989.
- [13] E. Hernández, D. O'Regan, *On a new class of abstract impulsive differential equations*, Proc. Amer. Math. Soc., **141**(2013), 1641-1649.
- [14] A.A. Kilbas, S.A. Marzan, *Nonlinear differential equations with the Caputo fractional derivative in the space of continuously differentiable functions*, Differential Eq., **41**(2005), 84-89.
- [15] A.A. Kilbas, H.M. Srivastava, J.J. Trujillo, *Theory and Applications of Fractional Differential Equations*, Elsevier Science B.V., Amsterdam, 2006.
- [16] K.S. Miller, B. Ross, *An Introduction to the Fractional Calculus and Differential Equations*, John Wiley, New York, 1993.
- [17] A. Pazy, *Semigroups of Linear Operators and Applications to Partial Differential Equations*, Springer-Verlag, New York, 1983.
- [18] M. Pierri, D. O'Regan, V. Rolnik, *Existence of solutions for semi-linear abstract differential equations with not instantaneous impulses*, Appl. Math. Comput., **219**(2013), 6743-6749.
- [19] I. Podlubny, I. Petráš, B.M. Vinagre, P. O'Leary, L. Dorčák, *Analogue realizations of fractional-order controllers. fractional order calculus and its applications*, Nonlinear Dynam., **29**(2002), 281-296.
- [20] I.A. Rus, *Ulam stabilities of ordinary differential equations in a Banach space*, Carpathian J. Math., **26**(2010), 103-107.
- [21] S.G. Samko, A.A. Kilbas, O.I. Marichev, *Fractional Integrals and Derivatives. Theory and Applications*, Gordon and Breach, Yverdon, 1993.
- [22] A.N. Vityuk, A.V. Golushkov, *Existence of solutions of systems of partial differential equations of fractional order*, Nonlinear Oscil., **7**(2004), no. 3, 318-325.
- [23] J. Wang, M. Fečkan, Y. Zhou, *Ulam's type stability of impulsive ordinary differential equations*, J. Math. Anal. Appl., **395**(2012), 258-264.
- [24] J. Wang, Y. Zhang, *Existence and stability of solutions to nonlinear impulsive differential equations in  $\beta$ -normed spaces*, Electron. J. Differential Eq., **2014**(2014), no. 83, 1-10.
- [25] Y. Zhou, F. Jiao, *Nonlocal Cauchy problem for fractional evolution equations*, Nonlinear Anal., Real World Appl., **11**(2010), 4465-4475.
- [26] Y. Zhou, *Basic Theory of Fractional Differential Equations*, World Scientific, Singapore, 2014.

*Received: April 24, 2014; Accepted: July 17, 2014.*

