

APPROXIMATE CONTROLLABILITY OF FRACTIONAL NEUTRAL DIFFERENTIAL SYSTEMS WITH BOUNDED DELAY

FANG WANG*** AND ZHENGAN YAO*

* School of Mathematics and Computational Science, Sun Yat-Sen University
Guangdong Prov., P.R. China
and

** School of Mathematics and Statistics, Changsha University of Science and Technology
Changsha 410114, Hunan Prov., P.R.China

E-mail: 46096140@qq.com

Abstract. In this paper, by using fractional power of operators and Schauder fixed point theorem, we study the approximate controllability of fractional neutral differential systems with bounded delay. The existence and uniqueness of mild solution of the system is also proved and an example is given to illustrate the theory.

Key Words and Phrases: Fractional neutral differential systems, Schauder fixed point theorem, compact semigroup, approximate controllability.

2010 Mathematics Subject Classification: 34H05, 47H10.

Acknowledgments. The author is highly grateful for the referees careful reading and comments on this paper. The first author is supported by the NSFC Granted 11526038, 11301039, 11301040. The second author is supported by the NSFC Granted 11431015, 11271381.

REFERENCES

- [1] R.P. Agarwal, M. Belmekki, M. Benchohra, *A survey on semilinear differential equations and inclusions involving Riemann-Liouville fractional derivative*, Adv. Differ. Eq., 2009, Article ID 981728.
- [2] N.U. Ahmed, *Dynamic Systems and Control with Applications*, Hackensack, NJ, World Scientific, 2006.
- [3] H.M. Ahmed, *Controllability of fractional stochastic delay equations*, Lobachevskii J. Math., **30**(2009), no. 3, 195-202.
- [4] H.M. Ahmed, *Boundary controllability of nonlinear fractional integrodifferential systems*, Adv. Difference Eq., 2010, Article ID 279493.
- [5] M. Benchohra, J. Henderson, S.K. Ntouyas, A. Quahab, *Existence results for fractional order functional differential equations with infinite delay*, J. Math. Anal. Appl., **338**(2008), 1340-1350.
- [6] L. Debanth, *Recents applications of fractional calculus to science and engineering*, Int. J. Math. Appl. Sci., **54**(2003), 3413-3442.
- [7] A. Debbouche, D. Baleanu, *Controllability of fractional evolution nonlocal impulsive quasilinear delay integro-differential systems*, Comput. Math. Appl., **62**(2011), 1442-1450.

- [8] K. Diethelm, A.D. Freed, *On the solution of nonlinear fractional order differential equations used in the modeling of viscoelasticity*, in: Scientific Computing in Chemical Engineering II-Computational Fluid Dynamics, Reaction Engineering and Molecular Properties (F. Keil, W. Machens, H. Voss, J. Werther - Eds.), Springer-Verlag, Heidelberg, 1999.
- [9] M.M. El-Borai, *Some probability densities and fundamental solutions of fractional evolution equations*, Chaos Solitons & Fractals, **14**(2002), 433-440.
- [10] H.R. Hernández, E. Hernández, *Approximate controllability of second-order distributed implicit functional systems*, Nonlinear Anal., **70**(2009), 1023-1039.
- [11] E. Hernández, D. O'Regan, K. Balachandran, *On recent developments in the theory of abstract differential equations with fractional derivatives*, Nonlinear Anal., **73**(2010), 3462-3471.
- [12] R. Hilfer, *Applications of Fractional Calculus in Physics*, World Scientific, Singapore, 2000.
- [13] A.A. Kilbas, H.M. Srivastava, J.J. Trujillo, *Theory and Applications of Fractional Differential Equations*, North-Holland Math. Studies, vol. 204, B.V. Amsterdam, Elsevier Science, 2006.
- [14] S. Kumar, N. Sukavanam, *Approximate controllability for a class of semilinear delay control system of fractional order*, Nonlinear Stud., **20**(2013), 73-83.
- [15] S. Kumar, N. Sukavanam, *On the approximate controllability of fractional order control system with delay*, Nonlinear Dynamics and System Theory, **13**(2013), 69-78.
- [16] S. Kumar, N. Sukavanam, *Approximate controllability of fractional order semilinear systems with bounded delay*, J. Diff. Eq., **252**(2012), 6163-6174.
- [17] S. Kumar, N. Sukavanam, *Approximate controllability of fractional order neutral control systems with delay*, Intern. J. Nonlinear Science, **13**(2012), 454-462.
- [18] M. A. Mckibben, *Approximate controllability of second-order functional evolution equations*, J. Optim. Theory Appl., **117**(2003), no. 2, 397-414.
- [19] K.S. Miller, B. Ross, *An Introduction to Fractional Calculus and Fractional Differential Equations*, New York, Wiley, 1999.
- [20] K. Naito, *Controllability of semilinear control systems dominated by the linear part*, SIAM J. Control Optim., **25**(1987), no. 3, 715-722.
- [21] A. Pazy, *Semigroup of Linear Operators and Applications to Partial Differential Equations*, Berlin, Springer-Verlag, 1983.
- [22] L. Podlubny, *Fractional Differential Equations*, San Diego, Academic Press, 1999.
- [23] Yu. Rossikhin, M.V. Shitikova, *Applications of fractional calculus to dynamic problems of linear and nonlinear hereditary mechanics of solids*, Appl. Mech. Rev., **50**(1997), no. 1, 15-67.
- [24] R. Sakthivel, Y. Ren, N.I. Mahmudor, *On the approximate controllability of semilinear fractional differential systems*, Comput. Math. Appl., **62**(2011), 1451-1459.
- [25] N. Sukavanam, *Approximate controllability of semilinear control system with growing nonlinearity*, in: Math. Theory of Control Proc., Marcel Dekker, New York, 1993, 353-357.
- [26] N. Sukavanam, S. Kumar, *Approximately controllability of fractional order semilinear delay systems*, J. Optim. Theory Appl., **151**(2011), no. 2, 373-384.
- [27] N. Sukavanam, N. K. Tomar, *Approximate controllability for semilinear delay control systems*, Nonlinear Funct. Anal. Appl., **12**(2007), no. 1, 53-59.
- [28] N. Sukavanam, S. Tafesse, *Approximate controllability of a delayed semilinear control system with growing nonlinear term*, Nonlinear Anal., **74**(2011), 6868-6875.
- [29] Z. Tai, X. Wang, *Controllability of fractional-order impulsive neutral functional infinite delay integrodifferential systems in Banach spaces*, Appl. Math. Lett., **22**(2009), 1760-1765.
- [30] V.E. Tarasov, *Fractional Dynamics: Applications of Fractional Calculus to Dynamics of Particles, Fields and Media*, HEP, Springer, 2010.
- [31] F. Wang, Z.H. Liu, J. Li, *Complete controllability of fractional neutral differential systems in abstract space*, Abstract Appl. Anal., 2013, Article ID 529025, 1-11.
- [32] Z. Yan, *Controllability of fractional-order partial neutral functional integro-differential inclusions with infinite delay*, J. Franklin Inst., **348**(2011), 2156-2173.
- [33] Y. Zhou, F. Jiao, *Existence of mild solutions for fractional neutral evolution equations*, Comput. Math. Appl., **59**(2010), 1063-1077.
- [34] Y. Zhou, F. Jiao, J. Li, *Existence and uniqueness for fractional neutral differential equations with infinite delay*, Nonlinear Anal., **71**(2009), 3429-3256.

Received: January 15, 2014; Accepted: May 19, 2014.

