

PARTIAL METRIC, FIXED POINTS, VARIATIONAL PRINCIPLES

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Abstract. The paper is mainly devoted to fixed point theorems. More general versions of important theorems of Matkowski, Romaguera, Takahashi, Brøndsted, and Banach are proved. Also, Ekeland's variational principle is extended. On the way to the main results, short proofs of crucial properties of partial metric spaces are proposed. In addition, the notion of kernel of partial metric is suggested together with a standard method of proving fixed point theorems. We also supply a useful criterion for Cauchy sequences.

Key Words and Phrases: Partial metric space, Cauchy sequence, fixed point, variational principle.

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