

## ON GLOBAL EXISTENCE OF CERTAIN IMPULSIVE MULTI-ORDERS FRACTIONAL DIFFERENTIAL PROBLEM

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**Abstract.** In this paper, we introduce a novel approach to tackle a class of fractional differential problems with impulses on the positive half-ray. So, we establish the existence of a bounded solution to certain impulsive multi-orders fractional initial value problem in a finite dimensional Banach space. The obtained result is based on the Schauder's fixed point theorem as well as certain continuation process. Finally, an illustrative example is provided.

**Key Words and Phrases:** Caputo's fractional derivative, multi-orders, impulsive conditions, Schauder's fixed point Theorem, PC-Ascoli-Arzela Theorem, continuation process.

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