# AROUND PEROV'S FIXED POINT THEOREM FOR MAPPINGS ON GENERALIZED METRIC SPACES

JACEK JACHYMSKI AND JAKUB KLIMA

Institute of Mathematics, Łódź University of Technology
Wólczańska 215, 93-005 Łódź, Poland
E-mail: jacek.jachymski@p.lodz.pl, jakub.klima.87@gmail.com

**Abstract.** We revisit Perov's fixed point theorem for selfmaps of a set endowed with a vector metric taking values in the Euclidean space $\mathbb{R}^m$. In particular, we show that this result is subsumed by the classical Banach contraction principle. We also obtain a generalization of Perov's theorem by considering mappings on $K$-metric spaces satisfying a nonlinear Lipschitz condition. Two applications are presented and some characterizations of convergence in $K$-metric spaces are given.

**Key Words and Phrases**: Generalized metric space, Perov's fixed point theorem, $K$-metric space, cone metric space, spectral radius, Cauchy initial value problem, solid cone, normal cone, nonlinear Lipschitz condition.

**2010 Mathematics Subject Classification**: 47H09, 47H10, 54H25, 34A12, 46B40.

## References

[1] C.D. Aliprantis, R. Tourky, *Cones and Duality*, Graduate Studies in Mathematics **84**, American Math. Soc., Providence, RI, 2007.

[2] R. Atkins, M.F. Barnsley, A. Vince, D.C. Wilson, *A characterization of hyperbolic affine iterated function systems*, Topology Proc., **36**(2010), 189–211.

[3] K. Deimling, *Nonlinear Functional Analysis*, Springer-Verlag, Berlin, 1985.

[4] E. De Pascale, L. De Pascale, *Fixed points for some non-obviously contractive operators*, Proc. Amer. Math. Soc., **130**(2002), 3249–3254.

[5] W.-S. Du, *A note on cone metric fixed point theory and its equivalence*, Nonlinear Anal., **72**(2010), 2259–2261.

[6] H. Dym, *Linear Algebra in Action*, Graduate Studies in Mathematics **78**, Amer. Math. Soc., Providence, RI, 2007.

[7] K. Goebel, *On a property of Lipschitzian transformations*, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys., **16**(1968), 27–28.

[8] K. Goebel, W.A. Kirk, *Topics in Metric Fixed Point Theory*, Cambridge Studies in Advanced Mathematics **28**, Cambridge University Press, Cambridge, 1990.

[9] L.-G. Huang, X. Zhang, *Cone metric spaces and fixed point theorems of contractive mappings*, J. Math. Anal. Appl., **332**(2007), 1468–1476.

[10] J. Jachymski, *Remetrization theorems for finite families of mappings and hyperbolic iterated function systems*, in "Infinite Products of Operators and Their Applications", Contemp. Math. 636, Amer. Math. Soc., Providence, R.I., 2015, 131-139.

[11] S. Janković, Z. Kadelburg, S. Radenović, *On cone metric spaces: a survey*, Nonlinear Anal., **74**(2011), 2591–2601.

[12] Z. Kadelburg, S. Radenović, V. Rakočević, *A note on the equivalence of some metric and cone metric fixed point results*, Appl. Math. Lett., **24**(2011), 370–374.

[13] M. Khani and M. Pourmahdian, *On the metrizability of cone metric spaces*, Topology Appl., **158**(2011), 190–193.

[14] M.A. Krasnosel'skiĭ, G.M. Vaĭnikko, P.P. Zabreĭko, Ya.B. Rutitskiĭ, V.Ya. Stetsenko, *Approximate Solution of Operator Equations*, Wolters-Noordhoff Publishing, Groningen, 1972.

[15] M. Kuczma, B. Choczewski, R. Ger, *Iterative Functional Equations*, Encyclopedia of Mathematics and its Applications **32**, Cambridge University Press, Cambridge, 1990.

[16] G. Kurepa, *Tableaux ramifiés d'ensembles. Espaces pseudo-distanciés*, C.R. Math. Acad. Sci. Paris, **198**(1934), 1563–1565.

[17] B. Lou, *Fixed points for operators in a space of continuous functions and applications*, Proc. Amer. Math. Soc., **127**(1999), 2259–2264.

[18] J. Matkowski, *Integrable solutions of functional equations*, Dissertationes Math. (Rozprawy Mat.), **127**(1975), 68 pp.

[19] I. Păvăloiu, *La résolution des systèmes d'équations opérationnelles à l'aide des méthodes itératives*, Mathematica (Cluj), **11**(34)(1969), 137–141.

[20] A.I. Perov, *On the Cauchy problem for a system of ordinary differential equations* (Russian), Približ. Metod. Rešen. Differencial. Uravnen. Vyp., **2**(1964), 115–134.

[21] A.I. Perov, A.V. Kibenko, *On a certain general method for investigation of boundary value problems* (Russian), Izv. Akad. Nauk SSSR Ser. Mat., **30**(1966), 249–264.

[22] A. Petruşel, G. Petruşel, C. Urs, *Vector-valued metrics, fixed points and coupled fixed points for nonlinear operators*, Fixed Point Theory Appl., **2013**, 2013:218, 21 pp.

[23] P.D. Proinov, *A unified theory of cone metric spaces and its applications to the fixed point theory*, Fixed Point Theory Appl., **2013**, 2013:103, 38 pp.

[24] Sh. Rezapour, R. Hamlbarani, *Some notes on the paper:"Cone metric spaces and fixed point theorems of contractive mappings" by L.-G. Huang and X. Zhang*, J. Math. Anal. Appl., **345**(2008), 719–724.

[25] I.A. Rus, *On the fixed points of mappings defined on a Cartesian product. II. Metric spaces* (Romanian), Stud. Cerc. Mat., **24**(1972), 897–904.

[26] I.A. Rus, A. Petruşel and G. Petruşel, *Fixed Point Theory*, Cluj University Press, Cluj-Napoca, 2008.

[27] P.P. Zabreĭko, *K-metric and K-normed linear spaces: survey*, Collect. Math., **48**(1997), 825–859.