

FIXED-POINT-FREE FUNCTIONS GENERATING COUNTEREXAMPLES TO ROLLE'S THEOREM IN ℓ_2

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Abstract. The map $T : \ell_2 \rightarrow \ell_2$ such that $T(x) = (\frac{1}{2} - \|x\|^2) \cdot e_1 + Rx$, as well as the map $N(x) = \sqrt{1 - \|x\|^2} \cdot e_1 + Rx$, were used by this author to produce two corresponding counterexamples to the classical Rolle's theorem in the closed unit ball of ℓ_2 . In this paper we introduce a class of maps, containing the before mentioned examples, which can be used to generate counterexamples to Rolle's theorem in the unit ball of ℓ_2 .

Key Words and Phrases: Fixed-point-free maps, Rolle's theorem counterexamples.

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