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NEW GENERALIZATION OF THE EXISTENCE OF EQUILIBRIUM FOR GENERALIZED GAME IN ABSTRACT CONVEX SPACE

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Abstract. The purpose of this paper is to establish a general existence of equilibrium for generalized game in abstract convex space, where the preference correspondence has unionly open lower section and the constraint correspondence is transfer open valued. New notions of $\mathscr{U}_A$-mapping and $\mathscr{U}_A$-majorized mapping are introduced, in which the lower sections are unionly open. We first prove some new fixed point theorems for set-valued mapping in noncompact abstract convex space. Next, we obtain two existence theorems of maximal element for $\mathscr{U}_A$-mapping and $\mathscr{U}_A$-majorized mapping. Lastly, we establish new equilibrium existence theorems for qualitative game and generalized game. Besides, we can get more general results than that in the recent literature.
Key Words and Phrases: Fixed point, maximal element, generalized game, $\mathscr{U}_A$-majorized mapping, abstract convex space.
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