

KRASNOSELSKII ALGORITHM FOR FIXED POINTS OF MULTIVALUED QUASI-NONEXPANSIVE MAPPINGS IN CERTAIN BANACH SPACES

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Abstract. Let K be a nonempty, closed and convex subset of a uniformly convex real Banach space E . Suppose that $T : K \rightarrow CB(K)$ is a multi-valued quasi-nonexpansive mapping. A Krasnoselskii-type iteration sequence $\{x_n\}$ is constructed and shown to be an approximate fixed point sequence of T , that is, $\lim_{n \rightarrow \infty} \text{dist}(x_n, Tx_n) = 0$ holds. Convergence theorems are also proved under appropriate additional conditions.

Key Words and Phrases: Multi-valued quasi-nonexpansive mapping, Hausdorff metric, *-non-expansive mapping, fixed point.

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