

SEMILINEAR EVOLUTION SYSTEMS WITH NONLINEAR CONSTRAINTS

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Abstract. The purpose of the present paper is to study the existence of solutions to semilinear evolution systems with nonlinear constraints. We establish new existence results using the fixed point principles of Perov and Schauder, combined with the technique that uses matrices with the spectral radius less than one and vector-valued norms. This vectorial approach is fruitful for the treating of systems in general and allows the system nonlinearities to behave independently as much as possible. Moreover, the constants from the Lipschitz or growth conditions are put into connection with the support of the nonlinear operators expressing the constraints. The paper extends and complements previous results from the literature.

Key Words and Phrases: Abstract evolution equation, nonlocal condition, fixed point, vector-valued norm, spectral radius of a matrix.

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