# EXISTENCE THEOREM FOR DIRICHLET PROBLEM FOR DIFFERENTIAL INCLUSION DRIVEN BY THE $p(x)$-LAPLACIAN

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**Abstract.** We study a class of second-order differential inclusion problem involving the $p(x)$-Laplacian. Using the techniques from multivalued analysis and the Leray-Schauder fixed point theorem, we establish the existence theorems under certain conditions.
**Key Words and Phrases**: Multifunction, maximal monotone operator, continuous selectors, differential inclusion, $p(x)$-Laplacian, Leray-Schauder fixed point theorem.
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