STRONG CONVERGENCE THEOREMS BY HYBRID METHOD FOR SEMIGROUPS OF NOT NECESSARILY CONTINUOUS MAPPINGS IN BANACH SPACES

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Abstract. In this paper, we establish a strong convergence theorem by the shrinking hybrid method for semigroups of not necessarily continuous mappings in Banach spaces. Using the result, we obtain well-known and new strong convergence theorems which are connected with results by the shrinking hybrid method in Hilbert spaces and Banach spaces.
Key Words and Phrases: Attractive point, Banach space, fixed point, generalized nonspreading mapping, hybrid method, invariant mean, semigroup, strongly asymptotically invariant net.
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REFERENCES

[1] Y. I. Alber, *Metric and generalized projections in Banach spaces: Properties and applications*, in Theory and Applications of Nonlinear Operators of Accretive and Monotone Type (A.G. Kartsatos - Ed.), Marcel Dekker, New York, 1996, pp. 15–50.
[2] Y. I. Alber, S. Reich, *An iterative method for solving a class of nonlinear operator equations in Banach spaces*, PanAmer. Math. J., **4**(1994), 39-54.
[3] S. M. Alsulami, N. Hussain, W. Takahashi, *Weak convergence theorems for semigroups of not necessarily continuous mappings in Banach spaces*, J. Convex Anal., **22**(2015), 81-100.
[4] S. Atsushiba, W. Takahashi, *Nonlinear ergodic theorems without convexity for nonexpansive semigroups in Hilbert spaces*, J. Nonlinear Convex Anal., **14**(2013), 209–219.
[5] J.-B. Baillon, *Un theoreme de type ergodique pour les contractions non lineaires dans un espace de Hilbert*, C.R. Acad. Sci. Paris Ser. A-B, **280**(1975), 1511-1514.
[6] J.-B. Baillon, H. Brezis, *Une remarque sur le comportement asymptotique des semigroupes non lineares*, Houston J. Math., **4**(1978), 1–9.

[7] M. M. Day, *Amenable semigroup*, Illinois J. Math., **1**(1957), 509–544.

[8] B. Halpern, *Fixed points of nonexpanding maps*, Bull. Amer. Math. Soc., **73**(1967), 957-961.

[9] N. Hirano, K. Kido, W. Takahashi, *Nonexpansive retractions and nonlinear ergodic theorems in Banach spaces*, Nonlinear Anal., **12**(1988), 1269-1281.

[10] T. Honda, T. Ibaraki, W. Takahashi, *Duality theorems and convergence theorems for nonlineaqr mappings in Banach spaces*, Int. J. Math. Statist., **6**(2010), 46–64.

[11] N. Hussain, W. Takahashi, *Weak and strong convergence theorems for semigroups of mappings without continuity in Hilbert spaces*, J. Nonlinear Convex Anal., **14**(2013), 769–783.

[12] T. Ibaraki, Y. Kimura, W. Takahashi, *Convergence theorems for generalized projections and maximal monotone operators in Banach spaces*, Abst. Appl. Anal., **2003**(2003), 621–629.

[13] T. Ibaraki, W. Takahashi, *A new projection and convergence theorems for the projections in Banach spaces*, J. Approx. Theory, **149**(2007), 1–14.

[14] T. Ibaraki, W. Takahashi, *Fixed point theorems for nonlinear mappings of nonexpansive type in Banach spaces*, J. Nonlinear Convex Anal., **10**(2009), 21–32.

[15] T. Ibaraki, W. Takahashi, *Generalized nonexpansive mappings and a proximal-type algorithm in Banach spaces*, Contemp. Math., **513**, Amer. Math. Soc., Providence, RI, 2010, pp. 169–180.

[16] T. Ibaraki, W. Takahashi, *Strong convergence theorems for finite generalized nonexpansive mappings in Banach spaces*, J. Nonlinear Convex Anal., **12**(2011), 407–428.

[17] W. Inthakon, S. Dhompongsa, W. Takahashi, *Strong convergence theorems for maximal monotone operators and generalized nonexpansive mappings in Banach spaces*, J. Nonlinear Convex Anal., **11**(2010), 45–63.

[18] S. Kamimura, W. Takahashi, *Strong convergence of a proximal-type algorithm in a Banach apace*, SIAM J. Optim., **13**(2002), 938–945.

[19] P. Kocourek, W. Takahashi, J.-C. Yao, *Fixed point theorems and weak convergence theorems for generalized hybrid mappings in Hilbert spaces*, Taiwanese J. Math., **14**(2010), 2497–2511.

[20] P. Kocourek, W. Takahashi, J. -C. Yao, *Fixed point theorems and ergodic theorems for nonlinear mappings in Banach spaces*, Adv. Math. Econ., **15**(2011), 67–88.

[21] F. Kohsaka, W. Takahashi, *Generalized nonexpansive retractions and a proximal-type algorithm in Banach spaces*, J. Nonlinear Convex Anal., **8**(2007), 197-209.

[22] F. Kohsaka, W. Takahashi, *Existence and approximation of fixed points of firmly nonexpansive-type mappings in Banach spaces*, SIAM J. Optim., **19**(2008), 824-835.

[23] F. Kohsaka, W. Takahashi, *Fixed point theorems for a class of nonlinear mappings related to maximal monotoneoperators in Banach spaces*, Arch. Math. (Basel), **91**(2008), 166-177.

[24] L.J. Lin, W. Takahashi, *Attractive point theorem for nonspreading mappings in Banach space*, J. Convex Anal., **20**(2013), 265–284.

[25] W.R. Mann, *Mean value methods in iteration*, Proc. Amer. Math. Soc., **4**(1953), 506-510.

[26] U. Mosco, *convergence of convex sets and of solutions of variational inequalities*, Adv. Math., **3**(1969), 510–585.

[27] K. Nakajo, W. Takahashi, *Strong convergence theorems for nonexpansive mappings and nonexpansive semigroups*, J. Math. Anal. Appl., **279**(2003), 372–379.

[28] Z. Opial, *Weak convergence of the sequence of successive approximations for nonexpansive mappings*, Bull. Amer. Math. Soc., **73**(1967), 591–597.

[29] S. Reich, *A weak convergence theorem for the alternative method with Bregman distance*, in Theory and Applications of Nonlinear Operators of Accretive and Monotone Type (A.G. Kartsatos - Ed.), Marcel Dekker, New York, 1996, pp. 313-318.

[30] W. Takahashi, *A nonlinear ergodic theorem for an amenable semigroup of nonexpansive mappings in a Hilbert space*, Proc. Amer. Math. Soc., **81**(1981), 253–256.

[31] W. Takahashi, *Nonlinear Functional Analysis*, Yokohama Publ., Yokohama, 2000.

[32] W. Takahashi, *Convex Analysis and Approximation of Fixed Points (Japanese)*, Yokohama Publishers, Yokohama, 2000.

[33] W. Takahashi, *Fixed point theorems for new nonexpansive mappings in a Hilbert space*, J. Nonlinear Convex Anal., **11**(2010),79-88.

[34] W. Takahashi, Y. Takeuchi, R. Kubota, *Strong convergence theorems by hybrid methods for families of nonexpansive mappings in Hilbert spaces*, J. Math. Anal. Appl., **341**(2008), 276–286.

[35] W. Takahashi, N.-C. Wong, J.-C. Yao, *Attractive point and mean convergence theorems for semigroups of mappings without continuity in Hilbert spaces*, J. Nonlinear Convex Anal., **15**(2014), 1087-1103.

[36] W. Takahashi, N.-C. Wong, J.-C. Yao, *Attractive point and mean convergence theorems for semigroups of mappings without continuity in Banach spaces*, J. Fixed Point Theory Appl., **16**(2014), 203-227.

[37] W. Takahashi, J. C. Yao, *Nonlinear operators of monotone type and convergence theorems with equilibrium problems in Banach spaces*, Taiwanese J. Math., **15**(2011), 787–818.