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FIXED POINT THEOREMS FOR MEIR-KEELER TYPE CONTRACTIONS IN METRIC SPACES

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Abstract. We establish a simple and powerful lemma that provides a criterion for sequences in metric spaces to be Cauchy. Using the lemma, it is then easily verified that the Picard iterates $\{T^n x\}$, where T is a contraction or asymptotic contraction of Meir-Keeler type, are Cauchy sequences. As an application, new and simple proofs for several known results on the existence of a fixed point for continuous and asymptotically regular self-maps of complete metric spaces satisfying a contractive condition of Meir-Keeler type are derived. These results include the remarkable fixed point theorem of Proinov in [Petko D. Proinov, Fixed point theorems in metric spaces, Nonlinear Anal. **46** (2006) 546–557], the fixed point theorem of Suzuki for asymptotic contractions in [Tomonari Suzuki, A definitive result on asymptotic contractions, J. Math. Anal. Appl. **335** (2007) 707–715], and others. We also prove some new fixed point theorems.
Key Words and Phrases: Meir-Keeler contractions, asymptotic contractions, fixed point theorems, complete metric spaces.
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References

[1] I.D. Arandelovic, *On a fixed point theorem of Kirk*, J. Math. Anal. Appl., **301**(2005), 384–385.
[2] S. Banach, *Sur les operations dans les ensembles abstraits et leur application aux equations integrales*, Fund. Math., **3**(1922), 133–181.
[3] R.M.T. Bianchini, *Su un problema di S. Reich riguardante la teoria dei punti fissi*, Boll. Un. Mat. Ital., **5**(1972), 103–108.
[4] D.W. Boyd, J.S. Wong, *On nonlinear contractions*, Proc. Amer. Math. Soc., **20**(1969), 458–469.
[5] F.E. Browder, W.V. Petryshyn, *The solution by iteration of nonlinear functional equations in Banach spaces*, Bull. Amer. Math. Soc., **72**(1966), 571–575.
[6] J. Caristi, *Fixed point theorems for mappings satisfying inwardness conditions*, Trans. Amer. Math. Soc., **215**(1976), 241–251.
[7] Y.J. Cho, P.P. Murthy, G. Jungck, *A theorem of Meer-Keeler type revisited*, Int. J. Math. Math. Sci., **23**(2000) 507–511.
[8] S.K. Chatterjea, *Fixed-point theorems*, C.R. Acad. Bulgare Sci., **25**(1972), 727–730.

[9] Lj. B. Ćirić, *Generalized contractions and fixed-point theorems*, Publ. Inst. Math. (Beograd) N.S., **12**(26)(1971), 19–20.
[10] Lj. B. Ćirić, *A generalization of Banach's contraction principle*, Proc. Amer. Math. Soc., **45**(1974), 267–273.
[11] Lj. B. Ćirić, *A new fixed-point theorem for contractive mappings*, Publ. Inst. Math. (N.S), **30**(44)(1981), 25–27.
[12] M. Edelstein, *On fixed and periodic points under contractive mappings*, J. London Math. Soc., **37**(1962), 74–79.
[13] I. Ekeland, *On the variational principle*, J. Math. Anal. Appl., **47**(1974), 324–353.
[14] M.A. Geraghty, *On contractive mappings*, Proc. Amer. Math. Soc., **40**(1973), 604–608.
[15] G.E. Hardy, T.D. Rogers, *A generalization of a fixed point theorem of Reich*, Canad. Math. Bull., **16**(1973), 201–206.
[16] J. Jachymski, *Equivalent conditions and the Meir-Keeler type theorems*, J. Math. Anal. Appl., **194**(1995), 293–303.
[17] R. Kannan, *Some results on fixed points. II*, Amer. Math. Monthly, **76**(1969), 405–408.
[18] W.A. Kirk, *Fixed points of asymptotic contractions*, J. Math. Anal. Appl., **277**(2003), 645–650.
[19] M. Kuczma, B. Choczewski, R. Ger, *Iterative Functional Equations, Encyclopedia of Mathematics and Applications*, vol. 32, Cambridge University Press, Cambridge, 1990.
[20] S. Leader, *Equivalent Cauchy sequences and contractive fixed points in metric spaces*, Studia Math., **76**(1983), 63–67.
[21] M. Maiti, T.K. Pal, *Generalization of two fixed point theorems*, Bull. Calcutta Math. Soc., **70**(1978), 57–61.
[22] J. Matkowski, *Integrable solutions of functional equations*, Diss. Math., **127**(1975).
[23] J. Matkowski, *Fixed point theorems for contractive mappings in metric spaces*, Cas. Pest. Mat., **105**(1980), 341–344.
[24] A. Meir, E. Keeler, *A theorem on contraction mappings*, J. Math. Anal. Appl., **28**(1969), 326–329.
[25] S. Park, B.E. Rhoades, *Meir-Keeler type contractive conditions*, Math. Japon., **26**(1)(1981), 13–20.
[26] Petko D. Proinov, *Fixed point theorems in metric spaces*, Nonlinear Anal., *64*(2006), 546–557.
[27] I.H.N. Rao, K.P.R. Rao, *Generalizations of fixed point theorems of Meir and Keeler type*, Indian J. Pure Appl. Math., **16**(1)(1985), 1249–1262.
[28] B.E. Rhoades, *A comparison of various definitions of contractive mappings*, Trans. Amer. Math. Soc., **226**(1977), 257–290.
[29] V.M. Sehgal, *On fixed and periodic points for a class of mappings*, J. London Math. Soc., **5**(2)(1972), 571–576.
[30] S.P. Singh, *Some results on fixed point theorems*, Yokahama Math. J., **17**(1969), 61–64.
[31] P.V. Subrahmanyam, *Remarks on some fixed point theorems related to Banach's contraction principle*, J. Math. Phys. Sci., *8*(1974), 445–457.
[32] T. Suzuki, *Fixed-point theorem for asymptotic contractions of Meir-Keeler type in complete metric spaces*, Nonlinear Anal., **64**(2006), 971–978.
[33] T. Suzuki, *A definitive result on asymptotic contractions*, J. Math. Anal. Appl., **335**(2007), 707–715.
[34] T. Suzuki, *A new type of fixed point theorem in metric spaces*, Nonlinear Anal., **71**(2009), 5313–5317.

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