REMARKS ON A LASALLE CONJECTURE ON GLOBAL ASYMPTOTIC STABILITY

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Abstract. In this paper we present some remarks on the following problem: Let $X$ be a (real or complex) Banach space, $\Omega \subset X$ be an open convex subset and $f : \Omega \to \Omega$ be an operator. We suppose that: (i) $f \in \mathcal{C}^1(X,X)$; (ii) the differential of $f$ at $x$, $df(x) : X \to X$ is a Picard operator for all $x \in \Omega$; (iii) the fixed point set of $f$, $F_f \neq \emptyset$. The problem is in which conditions $f$ is a Picard operator? In the case $X = \mathbb{R}^m$ or $X = \mathbb{C}^m$, this problem is in connection with a LaSalle Conjecture (J.P. LaSalle, The Stability of Dynamical Systems, SIAM, No. 25, 1976) and with the Belitskii-Lyubich Conjecture (G.R. Belitskii and Yu.I. Lyubich, Matrix Norms and their Applications, Birkhäuser, 1988).

We also formulate the following conjecture:

Let $X$ be a Banach space, $\Omega \subset X$ be an open convex subset and $f : \Omega \to \Omega$ be an operator. We suppose that: (i) $f \in \mathcal{C}^1(\Omega, X)$; (ii) $df^k(x)$ is a Picard operator, $\forall x \in \Omega, \forall k \in \mathbb{N}^*$; (iii) $F_f \neq \emptyset$. Then $f$ is a Picard operator.

Some research directions are also presented.

Key Words and Phrases: Banach space, differentiable nonlinear operator, fixed point, iterate, spectral radius, global asymptotic stability, Picard operator, LaSalle Conjecture, Belitskii-Lyubich Conjecture, discrete Markus-Yamabe Conjecture, Ostrowski property, stability under operator perturbation.

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REFERENCES


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