

## STRONG CONVERGENCE ALGORITHM FOR HIERARCHICAL FIXED POINT PROBLEMS OF A FINITE FAMILY OF NONEXPANSIVE MAPPINGS

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**Abstract.** In this paper, we propose an iterative algorithm for hierarchical fixed point problems of a finite family of nonexpansive mappings in the setting of real Hilbert spaces. We prove that the sequence generated by the proposed method algorithm converges strongly to a fixed point of a finite family of nonexpansive mappings which is also the solution of a variational inequality. Numerical examples are presented to illustrate the proposed method and convergence result. The iterative algorithm and results presented in this paper generalize, unify and improve the previously known results of this area.

**Key Words and Phrases:** Hierarchical fixed point problem, fixed point problem, nonexpansive mapping, averaged mapping.

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