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Δ-CONVERGENCE AND W-CONVERGENCE OF THE MODIFIED MANN ITERATION FOR A FAMILY OF ASYMPTOTICALLY NONEXPANSIVE TYPE MAPPINGS IN COMPLETE CAT(0) SPACES

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Abstract. In this paper, we show Δ-convergence and w-convergence (in the sense of Ahmadi Kakavandi and Amini [2]) of modified Mann iteration

$$x_{n+1} = \alpha_n Py_n \oplus (1-\alpha_n)T_n^n Py_n,\ d(y_n, x_n) \leq e_n,\ x_0 \in C,$$

to a common fixed point of the sequence (T_n) of asymptotically nonexpansive type selfmappings on a closed and convex subset C of a complete CAT(0) space X, where $(\alpha_n) \subset [0,1]$, $(e_n) \subset \mathbb{R}^+$ and P is the nearest point projection on C. Our results extend the results in [16, 21] in the setting of complete CAT(0) spaces.

Key Words and Phrases: w-convergence, Δ-convergence, Asymptotically nonexpansive type selfmapping, Fixed point, CAT(0) space.

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