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ESSENTIAL SETS OF FIXED POINTS FOR CORRESPONDENCES WITH APPLICATION TO NASH EQUILIBRIA

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Abstract. This paper studies essential stabilities of fixed points for correspondences. The existence of minimal essential sets of fixed points is proved under the perturbation of correspondences and domains. We show that a kind of minimal essential set is connected. As an application, the existence of minimal essential sets of Nash equilibria is deduced, and these sets can resist the dual perturbations of best responses and strategy sets for a noncooperative game.
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