

SOME FIXED POINT THEOREMS OF A KRASNOSEL'SKII TYPE AND APPLICATION TO DIFFERENTIAL INCLUSIONS

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Abstract. In this paper, we establish some fixed point results of Krasnosel'skii type fixed point theorem for the sum of $B + G$, where B is expansive linear operator and G is a weakly continuous map or weakly-weakly upper semi-continuous multivalued operator. Finally, our results are used to prove the existence of solution for multivalued Dirihlet problem in reflexive Banach spaces.

Key Words and Phrases: Multivalued map, compact set, weakly compact, fixed point, expansive.

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