

## BEST PROXIMITY PAIR THEOREMS FOR NONCYCLIC MAPPINGS IN BANACH AND METRIC SPACES

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**Abstract.** Let  $A$  and  $B$  be two nonempty subsets of a metric space  $X$ . A mapping  $T : A \cup B \rightarrow A \cup B$  is said to be noncyclic if  $T(A) \subseteq A$  and  $T(B) \subseteq B$ . For such a mapping, a pair  $(x, y) \in A \times B$  such that  $Tx = x$ ,  $Ty = y$  and  $d(x, y) = \text{dist}(A, B)$  is called a best proximity pair. In this paper we give some best proximity pair results for noncyclic mappings under certain contractive conditions.

**Key Words and Phrases:** Best proximity pair; noncyclic contraction, noncyclic contraction in the sense of Kannan; noncyclic contraction in the sense of Chatterjea, reflexive metric space.

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