

ON THE CHARACTERIZATION OF PARTIAL METRICS AND QUASIMETRICS

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Abstract. We present the relationship between the notion of partial metric, which has applications in Computer Science, that of quasimetric (which lacks symmetry) and that of standard metric. In this process the nonexpansive functions play an important role. We give some simple formulations of the sequence convergence and of the 0-completeness in partial metric spaces. We apply the results to the characterization of completeness in terms of Caristi's theorem in quasimetric spaces.

Key Words and Phrases: metric spaces, partial metric, quasimetric, fixed points, nonexpansive mappings, Caristi's theorem.

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