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ULAM-HYERS STABILITY OF SINGULAR INTEGRAL EQUATIONS, VIA WEAKLY PICARD OPERATORS

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Abstract. In this paper we investigate the Ulam-Hyers stability of several integral equations with singularity. First we give some results concerning the Ulam-Hyers stability of integral equations with weak singularities. Our approach is also suitable for studying some fractional differential equations. In order to emphasize this aspect we prove that some conditions (5) in S. Abbas, M. Benchohra, *Ulam-Hyers stability for the Darboux problem for partial fractional differential and integro-differential equations via Picard operators* published in Results Math. **65**(2014), 67-79 (respectively condition (3.1) from S. Abbas, M. Benchohra, A. Petruşel, *Ulam stability for partial fractional differential inclusions via Picard operators theory*, Electron. J. Qual. Theory Differ. Equ., 2014, No. 51, 1-13) can be omitted without losing the validity of the obtained results. In the second part we establish some generalized Ulam-Hyers-Rassias stability results for the Bessel equation and related equations. Our approach is based on fixed point methods and the obtained results are more general than those established by Byungbae Kim and Soon-Mo Jung in *Bessel's differential equation and its Hyers-Ulam stability* appeared in J. Inequal. Appl., Volume 2007.

Key Words and Phrases: Ulam-Hyers stability, Picard operators, Bessel equation, integral equations with singularities, fractional differential equations.
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