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CRITICAL KRASNOSELSKII-SCHAEFER TYPE FIXED POINT THEOREMS FOR WEAKLY SEQUENTIALLY CONTINUOUS MAPPINGS AND APPLICATION TO A NONLINEAR INTEGRAL EQUATION

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Abstract. In this paper, we first state some new fixed point theorems for operators of the form $A + B$ on a bounded closed convex set of a Banach space, where A is a weakly compact and weakly sequentially continuous mapping and B is either a weakly sequentially continuous nonlinear contraction or a weakly sequentially continuous separate contraction mapping. Second, we study the fixed point property for a larger class of weakly sequentially continuous mappings under weaker assumptions and we explore this kind of generalization by looking for the multivalued mapping $(I - B)^{-1}A$, when $I - B$ may not be injective. To attain this goal, we extend H. Schaefer's theorem to multivalued mappings having weakly sequentially closed graph. Our results generalize many known ones in the literature, in particular those obtained by C. Avramescu (2004, Electron. J. Qual. Theory Differ. Equ., $17, 1 - 10$), C. S. Barroso (2003, Nonlinear Anal., $55, 25 - 31$), T.A. Burton (1998, Appl. Math. Lett., $11, 85 - 88$), Y. Liu and Z. Li (2006, J. Math. Anal. Appl., $316, 237 - 255$ and 2008, Proc. Amer. Math. Soc., $316, 1213-1220$), H. Schaefer (1955, Math. Ann., $129, 415-416$) and M.A. Taoudi (2010, Nonlinear Anal., 72 $(1), 478 - 482$). Finally, we use our abstract results to derive an existence theory for an integral equation in a reflexive Banach space.
Key Words and Phrases: Krasnoselskii type fixed point theorem, Schaefer type fixed point theorem, nonlinear contraction mapping, De Blasi measure of weak noncompactness.
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