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A MULTIPARAMETER GLOBAL BIFURCATION THEOREM WITH APPLICATION TO A FEEDBACK CONTROL SYSTEM

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Abstract. In this paper, applying the method of integral guiding functions, we consider a multiparameter global bifurcation problem for periodic solutions of first order operator-differential inclusions whose multivalued parts are not necessarily convex-valued. It is shown how the abstract result can be applied to study the global structure of periodic solutions of a feedback control system with a two-dimensional parameter.

Key Words and Phrases: Global bifurcation, guiding function, operator-differential inclusion, periodic solution, fixed point.

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