

FIXED POINT THEOREMS FOR 1-SET CONTRACTIONS IN BANACH SPACES

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Abstract. This work presents some new fixed point theorems for 1-set contractions in Banach spaces. We first prove an existence and uniqueness result for a 1-set contraction mapping f when $I - f$ is ψ -expansive, extending [13, Proposition 3.4]. More generally, when $I - f$ is α - ψ -expansive, an existence result for 1-set contraction mappings is obtained. We then derive several fixed point results for the sum (in Banach spaces) and the product (in Banach algebras) of two operators, one of them is completely continuous and the other one is a 1-set contraction. In this context, the Furi-Pera boundary condition is investigated and comparison with recent results is given. Finally, [13, Proposition 3.4] is obtained as a consequence of a compactness result proved in [8]. The proofs essentially use the properties of the Kuratowski measure of noncompactness.

Key Words and Phrases: Nonexpansive map, 1-set contraction, ψ -expansive map, α - ψ -expansive map, Kuratowski measure of noncompactness, fixed point theorem, sum of operators, Banach algebra.

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