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ITERATIVE ALGORITHM FOR ZEROS OF BOUNDED MULTI-VALUED ACCRETIVE OPERATORS

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Abstract. Let E be a uniformly smooth real Banach space and $A : E \to 2^E$ a multi-valued mapping. An efficient iteration algorithm for approximating zeros of A, in the case that A is m-accretive and bounded, is studied and the sequence of the algorithm is proved to converge strongly to a point in $A^{-1}(0)$. We achieve this by using the celebrated result of Simeon Reich.
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