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WEAK ORTHOGONALITY AND SUZUKI NONEXPANSIVE-TYPE MAPPINGS

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Abstract. It is shown that if X is a weakly orthogonal Banach lattice, K is a nonempty weakly compact and convex subset of X and $T : K \to K$ satisfies condition (C) or is continuous and satisfies condition (C_λ) for some $\lambda \in (0, 1)$, then T has a fixed point. This generalizes Sims's result from [11].
Key Words and Phrases: Nonexpansive mapping, fixed point, weakly orthogonal lattice, mapping satisfying condition (C).
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