

GENERALIZED KY FAN MINIMAX INEQUALITIES FOR SET-VALUED MAPPINGS

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Abstract. In this paper, by virtue of the Ky Fan section theorem, the Kakutani-Fan- Glicksberg fixed point theorem and two nonlinear scalarization functions, we investigate some generalized Ky Fan minimax inequalities for set-valued mappings, in which the minimization and the maximization of set-valued mappings are taken in the sense of vector optimization.

Key Words and Phrases: Ky Fan minimax inequality, set-valued mapping, nonlinear scalarization function, fixed point theorem, vector optimization.

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