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TWO REMARKS ON THE MODIFIED HALPERN ITERATIONS IN CAT(0) SPACES

SATIT SAEJUNG*,**

*Department of Mathematics, Faculty of Science, Khon Kaen University
Khon Kaen 40002, Thailand
E-mail: saejung@kku.ac.th

**The Centre of Excellence in Mathematics, Commission on Higher Education (CHE)
Sri Ayudthaya Road, Bangkok 10400, Thailand

Abstract. The purpose of this paper is to give a short and simple proof of a generalization of Suzuki's lemma [9] in metric spaces of hyperbolic type as well as to include some remarks on the strong convergence theorem of modified Halpern iteration in CAT(0) spaces, which is proved by Cuntavepanit and Panyanak [4].
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